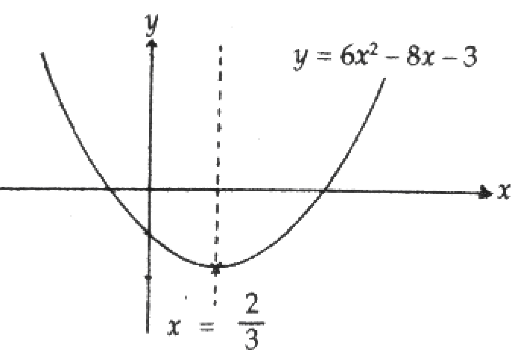
1995

1. (a) Given $3x+2y-1=0$, find the range of values of *x* if *y* < 5.
   (b) Find the range of values of *n* if $2n^2+n\geq 1$.

(a) $x > -3$ (b) $n \leqslant -1,\ n \geqslant \frac{1}{2}$

2. Without using the differentiation method, or drawing a graph, find the maximum or minimum value of the function $y=2(3x-1)(x+1)-12x-1$. Hence, sketch the graph of the function *y*.

minimum value $= -\frac{17}{3}$



3. (a) Find the range of the values of *k* so that the equation $x^2+kx+2k-3=0$ does not have a root.
   (b) Prove that the roots of the equation $(1-p)x^2+x+p=0$ are real and negative if 0 < *p* < 1.

(a) $2 < k < 6$

1996

4. $f(x)=0$ is a quadratic equation which has two different roots -3 and *p*.
   (a) Write $f(x)$ in the form of $ax^2+bx+c=0$.
   (b) The curve $y=kf(x)$ cuts the y-axis at (0, 60). Given $p=5$, calculate
   (i) value of *k*,
   (ii) the coordinate of the maximum point of the curve without using the differentiation method or drawing a graph.

(a) $f(x) = x^2 + (3+p)x - 3p$
(b) (i) $k = -4$
(ii) Maximum point = (1, 64)

5. Find the range of the values of *x* if
   (a) $x(x+1)<2$
   (b) $\frac{-3}{1-2x}\geq x$

(a) $-2 < x < 1$
(b) $x \leqslant -1$ or $\frac{1}{2} < x \leqslant \frac{3}{2}$

1997

6. A quadratic function $f(x)=2[(x-m)^2+n]$, where *m* and *n* are constants has a minimum point $P(6t,3t^2)$.
   (a) Express *m* and *n* in terms of *t*.
   (b) If $t=1$, find the range of the values of *h* so that the equation $f(x)=k$ has real roots.

(a) $f(x) = 2[(x-m)^2+n]$
$= 2(x-m)^2 + 2n$
The minimum value of $f(x)$ is $2n$ when x = m.
That is minimum point is $(m, 2n)$.
P is $(6t, 3t^2)$.
Hence, $m = 6t$, $2n = 3t^2$
$n = \frac{3}{2}t^2$

(b) $t = 1$, $f(x) = k$, $m = 6$, $n = \dfrac{3}{2}$

$$2[(x - m)^2 + n] = k$$
$$2[(x - 6)^2 + \frac{3}{2}] = k$$
$$2(x^2 - 12x + 36 + \frac{3}{2}] = k$$
$$2x^2 - 24x + 75 - k = 0$$

If $f(x) = k$ has real roots

$$b^2 - 4ac \geqslant 0$$
$$(-24)^2 - 4(2)(75 - k) \geqslant 0$$
$$576 - 600 + 8k \geqslant 0$$
$$8k \geqslant 24$$
$$k \geqslant 3$$

7. Find the range of values of x if
   (a) $2(3x^2 - x) \le 1 - x$,
   (b) $4y - 1 = 5x$ and $2y > 3 + x$.

(a)
$$2(3x^2 - x) \leqslant 1 - x$$
$$6x^2 - 2x + x - 1 \leqslant 0$$
$$6x^2 - x - 1 \leqslant 0$$
$$(3x + 1)(2x - 1) \leqslant 0$$
hence $-\dfrac{1}{3} \leqslant x \leqslant \dfrac{1}{2}$

(b)
$$2y > 3 + x$$
$$2(\frac{1 + 5x}{4}) > 3 + x$$
$$1 + 5x > 6 + 2x$$
$$3x > 5$$
$$x > \frac{5}{3}$$

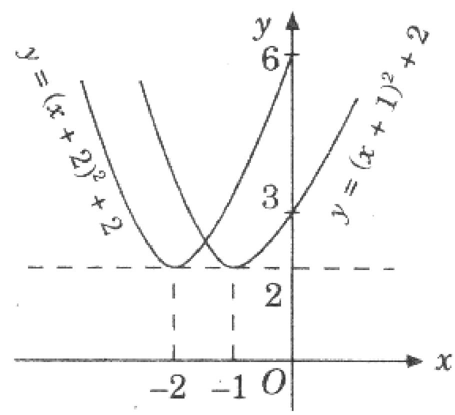
8. Given $y = x^2 + 2kx + 3k$ has a minimum value of 5.
   (a) Without using the differentiation method, find the two possible values of k.
   (b) With the values of k, sketch on the same axes, the two graphs for $y = x^2 + 2kx + 3k$.
   (c) State the coordinates of the minimum points of the graph $y = x^2 + 2kx + 6k$.

(a)
$$y = x^2 + 2kx + 3k$$
$$= x^2 + 2kx + k^2 - k^2 + 3k$$
$$= (x + k)^2 - k^2 + 3k$$

Minimum value $y = -k^2 + 3k$

$$\therefore \quad -k^2 + 3k = 2$$
$$-k^2 + 3k - 2 = 0$$
$$k^2 - 3k + 2 = 0$$
$$(k - 2)(k - 1) = 0$$
$$k = 2 \text{ or } k = 1$$

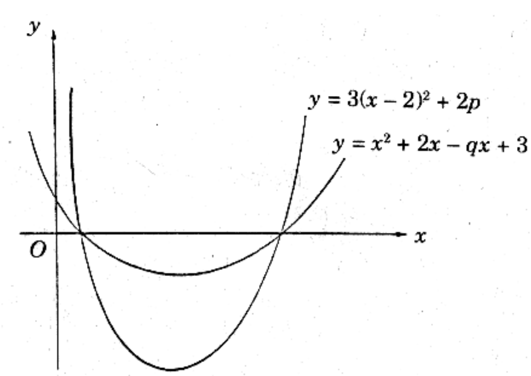
(b) If $k = 1$, $y = (x + 1)^2 + 2$
If $k = 2$, $y = (x + 2)^2 + 2$



(c) Minimum point $= (-k, -k^2 + 3k)$
$= (-k, 2)$

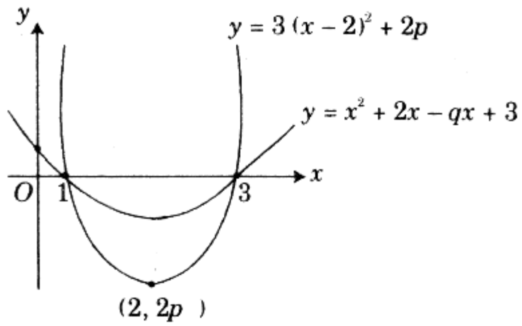
1998

9.



The diagram above shows the graph of a curve $y = x^2 + 2x - qx + 3$ and

$y = 3(x-2)^2 + 2p$ which intersect at two points on the x-axis. Find
(a) the values of p and q,
(b) the minimum values of the two curves.



(a) $y = x^2 + 2x - qx + 3 \quad \ldots ①$
$= x^2 + (2-q)x + 3$
$= x^2 + (2-q)x + \left(\frac{2-q}{2}\right)^2 - \left(\frac{2-q}{2}\right)^2 + 3$
$= \left[x + \left(\frac{2-q}{2}\right)\right]^2 - \left(\frac{2-q}{2}\right)^2 + 3$

Minimum point
$\left[-\left(\frac{2-q}{2}\right), -\left(\frac{2-q}{2}\right)^2 + 3\right]$

From $-\left(\frac{2-q}{2}\right) = 2$
$2 - q = -4$
$q = 2 + 4 = 6$

From ①, $y = x^2 + 2x - 6x + 3$
$= x^2 - 4x + 3$

When $y = 0$, $x^2 - 4x + 3 = 0$
$(x-1)(x-3) = 0$
$x - 1 = 0$ or $x - 3 = 0$
$x = 1$ or $x = 3$

$y = 3(x-2)^2 + 2p \quad \ldots ②$
Minimum point $(2, 2p)$
Substitute points $(1, 0)$ and $(3, 0)$ into ②,

$(1, 0)$, $0 = 3(1-2)^2 + 2p$
$= 3 + 2p$
$2p = -3$
$p = \frac{-3}{2}$

Or
$(3, 0)$, $0 = 3(3-2)^2 + 2p$
$= 3 + 2p$
$2p = -3$
$p = \frac{-3}{2}$

(b) Substitute $p = \frac{-3}{2}$ and $q = 6$ into the minimum points for both the curve
$(2, 2p) = \left[2, 2\left(\frac{-3}{2}\right)\right]$
$= (2, -3)$

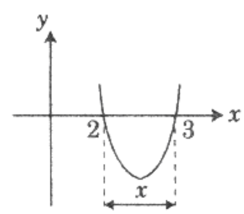
$\left[-\left(\frac{2-q}{2}\right), -\left(\frac{2-q}{2}\right)^2 + 3\right] = \left[-\left(\frac{2-6}{2}\right), -\left(\frac{2-6}{2}\right)^2 + 3\right]$
$= (2, -1)$

So, the minimum value for both the curve is -3 and -1.

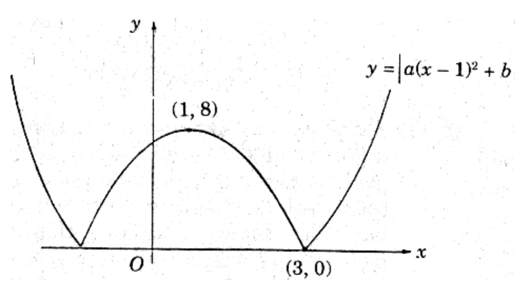
10.(a) Given $f(x) = 4x^2 - 1$. Find the range of values of x so that $f(x)$ is always positive.
(b) Find the range of values of x which satisfies the inequality $(x-2)^2 < (x-2)$.

(a) Given $f(x) = 4x^2 - 1$ and $f(x)$ is always positive
So $4x^2 - 1 > 0$
$(2x-1)(2x+1) > 0$
$2x - 1 > 0$ or $2x + 1 > 0$
$x > \frac{1}{2}$ or $x < -\frac{1}{2}$

(b) Given $(x-2)^2 < (x-2)$
Hence $x^2 - 4x + 4 < x - 2$
$x^2 - 4x + 4 - x + 2 < 0$
$x^2 - 5x + 6 < 0$
$(x-2)(x-3) < 0$
$2 < x < 3$

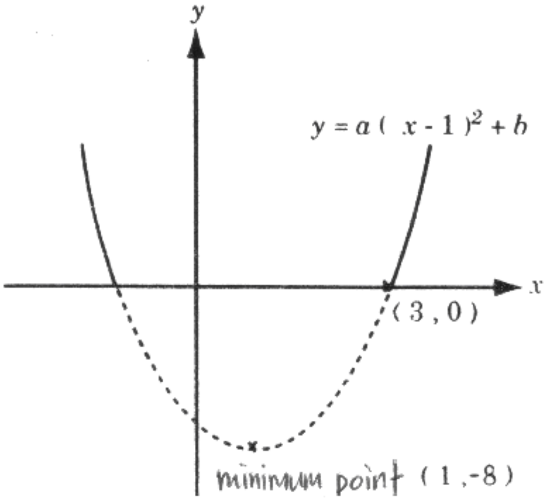


11.



The diagram shows the graph of a curve $y = \left|a(x-1)^2 + b\right|$. The point (1, 8) is the maximum point of the curve. Find
(a) the values of a and b,

(b) the range of values of y in the domain $1 \le x \le 4$.



(a) Using simultaneous equation

Point (3, 0)

$y = a(x-1)^2 + b$

$0 = a(3-1)^2 + b$

$0 = 4a + b$

$b = -4a$ ...... ①

Point (1, 8)

$y = a(x-1)^2 + b$

$-8 = a(1-1)^2 + b$

$b = -8$

Substitute $b = -8$ into ①

$-8 = -4a$

$a = 2$

(b) Equation is $y = |2(x-1)^2 - 8|$

For $x = 4$, $y = |2(4-1)^2 - 8|$

$= |2(3)^2 - 8|$

$= |18 - 8|$

$= |10|$

Range of y for domain $1 \le x \le 4$ is $0 \le y \le 10$.

1999

12.(a) Find the range of values of x so that $9 + 2x > 3$ and $19 > 3x + 4$.

(b) Given $2x + 3y = 6$, find the range of the values of x when $y < 4$.

(a) $9 + 2x > 3$

$x > -3$

$3x + 4 < 19$

$x < 5$

$\therefore -3 < x < 5$

(b) $3y = 6 - 2x$

$y = \dfrac{(6-2x)}{3}$

$y < 5$

$\dfrac{(6-2x)}{3} < 5$

$6 - 2x < 15$

$x > -\dfrac{9}{2}$

13.Find the range of x if given $(x-2)(2x+3) > (x-2)(x+2)$.

$(x-2)(2x+3) > (x-2)(x+2)$

$2x^2 - x - 6 > x^2 - 4$

$x^2 - x - 2 > 0$

$(x+1)(x-2) > 0$

$x < -1, x > 2$

2000

14.Without using the differentiation method or drawing a graph, find the maximum or minimum value of the function $y = 1 + 2x - 3x^2$. Hence, find the symmetrical axis of the graph of the function.

$y = 1 + 2x - 3x^2$

$= -3x^2 + 2x + 1$

$= -3(x^2 - \frac{2}{3}x) + 1$

$= -3[x^2 - \frac{2}{3}x + (-\frac{1}{3})^2 - (-\frac{1}{3})^2] + 1$

$= -3[(x - \frac{1}{3})^2 - \frac{1}{9}] + 1$

$= -3(x - \frac{1}{3})^2 + 1\frac{1}{3}$

Maximum value for $y = 1\frac{1}{3}$

The axis of symmetry is $x = \frac{1}{3}$

15. Find the range of the values of k so that the straight line $y = 2x + k$ does not intersect the curve $x^2 + y^2 - 6 = 0$.

$y = 2x + k$ .... ①

$x^2 + y^2 - 6 = 0$ .... ②

Substitute ① into ②,

$$x^2 + (2x + k)^2 - 6 = 0$$
$$x^2 + 4x^2 + 4xk + k^2 - 6 = 0$$
$$5x^2 + 4kx + k^2 - 6 = 0$$

Roots of two equation that does not intersect

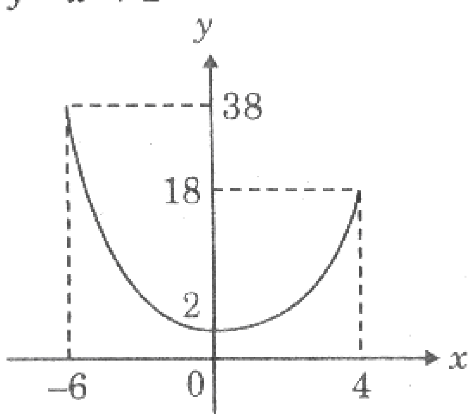
$$b^2 - 4ac < 0$$
$$16k^2 - 4(5)(k^2 - 6) < 0$$
$$16k^2 - 20k^2 + 120 < 0$$
$$-4k^2 < -120$$
$$k^2 > 30$$

$k < -5.477$ or $k > 5.477$

16. (a) Sketch the graph of $y = x^2 + 2$ for $-6 \le x \le 4$. Hence, state the range of values of y.

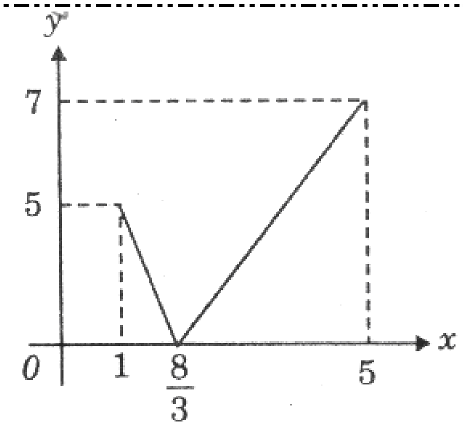
(b) Show that the function $3x - 2 - 2x^2$ is always negative for all real values of x.

(a) (i) $y = x^2 + 2$



The range of values of y, $2 \leqslant y \leqslant 38$

(ii)



The range of values of y, $0 \leqslant y \leqslant 7$

(b) $f(x) = 3x - 2 - 2x^2$
$= -2x^2 + 3x - 2$

If $f(x) = 0$

$$-2x^2 + 3x - 2 = 0$$
$$b^2 - 4ac = 9 - 4(-2)(-2)$$
$$= -7$$

Hence $b^2 - 4ac < 0$

There is no value of x when $f(x) = 0$. Hence the function $3x - 2 - 2x^2$ is always negative for all values of x.